

Finite Higher-Dimensional Unified Field Theory and TeV Physics

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Abstract

A unified field theory based on the compactification of a higher D -dimensional Einstein-Yang-Mills-Higgs action is developed. The extra $D - 4$ dimensions form a compact internal space with scale size R . An anomaly-free unified chiral model of quarks and leptons, described by $SO(18)$ in twelve dimensions, breaks down to $SO(10) \times SO(8) \rightarrow SO(10)$ with a non-trivial topological structure and three chiral families in four dimensions. A quantum field theory formalism in D -dimensions leads to a self-consistent, finite quantum gravity, Yang-Mills and Higgs theory, which is unitary and gauge invariant to all orders of perturbation theory. The gauge hierarchy problem is solved due to the exponential damping of the Higgs self-energy loop graph for energies greater than ~ 1 TeV, and because of the reduction of quantum gravity to a scale of several TeV. The compactification scale is $M_c \geq 1$ TeV, leading to Kaluza-Klein excitations and experimental signatures at a scale of several TeV. Various scenarios for evading fast proton decay are discussed.

1 Introduction

The standard model has been verified to remarkable accuracy[1] down to scales of 10^{-15} cm, corresponding to energies up to $\simeq 100$ GeV. With the discovery of the top quark with a mass $m_t = 175.6(5.5)$ GeV, all the required fermions in the standard model are now in place. The Higgs particle, which represents a vital missing element in the standard model, is yet to be discovered. When it is found, we could just declare that particle physics is closed. However, there are conceptual difficulties with the standard model, which point to new physics beyond it. A successful unified theory of gravity and the standard model should at least accomplish the following:

1. Resolve the gauge hierarchy problem. The gauge hierarchy (or 't Hooft naturalness) problem besets the Higgs sector. The standard model cannot naturally explain the relative smallness of the weak scale of mass, set by the Higgs mechanism at $M_{\text{WS}} \sim 250$ GeV.
2. Reduce the number of unknown parameters.
3. Explain the origin of the three fermion generations in the standard model.
4. Provide a mathematically consistent quantum gravity theory which leads to finite scattering amplitudes to all orders in perturbation theory.
5. Guarantee that the proton remains stable in accordance with the current experimental lower bound on its decay lifetime τ_p .

A leading candidate for a unified theory of the standard model and gravity has been superstring theory[2]. String theory bypasses the problem of ultraviolet divergences of gravity by replacing the fundamental point-like object by a string, an extended one-dimensional object. String compactifications lead to a myriad of possible vacuum states as models of low energy particle physics, but recent developments in duality and p-branes[3] have led to new possibilities for unification models. D-branes are associated with gauge fields living in their world volume[4, 5, 6]. The standard model gauge group would correspond to gauge fields living in the world volume of 3-branes. If the gauge group comes from open strings starting and ending on a set of p-branes, then the string scale M_s can be lowered much below the Planck mass scale, $M_{\text{Planck}} \sim 10^{19}$ GeV, by using the formula (for 3-branes): $M_s^4 = \alpha_{\text{GUT}} M_c^3 M_{\text{Planck}} / \sqrt{2}$, where M_c is the compactification scale. If $M_s \sim 1$ TeV, then predictions could be made that might be checked by the new generation of accelerators. Dienes et al.[7] have shown that a Kaluza-Klein orbifold reduction can lead to a gauge coupling unification at a much lower energy scale than the usual GUT scale with power law behaviour instead of the familiar logarithmic scaling behaviour. A more radical idea has been to reduce the Planck scale of gravity to the TeV energy region[8], thus resolving the hierarchy problem.

In previous work[9], the author developed a model based on the gauge group $\mathcal{G} = SO(3, 1) \times SU_c(3) \times SU(2) \times U(1)$ in four dimensions. No attempt was made to unify the standard model with gravity. The unknown parameters such as coupling constants and fermion masses were to be determined by

a relativistic, Schrödinger-type eigenvalue equation, using the perturbatively finite and unitary formalism of finite quantum field theory (FQFT)[10-18]. The stability of the proton was guaranteed, for quarks and leptons were not combined in the same irreducible fermion representation.

If, indeed, baryon number is conserved, then the small but non-zero matter content of the universe is simply a matter of the initial conditions, and its value cannot be explained within the standard domain of physics, which is not a satisfactory state of affairs. Moreover, it is difficult to find non-trivial solutions to the relativistic eigenvalue equation for the mass spectrum and the coupling constants. In view of this, it is tempting to pursue further the possibility of discovering a unified field theory of gravity and the gauge fields of the standard model.

In the following, we shall pursue such a possible theory based on a higher-dimensional unified field theory. In the early eighties there was a revival of attempts to build a unified theory using the idea of a Kaluza-Klein pure gravity theory in D -dimensions[19]. These ideas were abandoned when it was discovered that assuming a Riemannian geometry in D -dimensions and an internal compact space, the models failed to predict flavor chiral fermions in the four-dimensional theory[20]. Beginning with a spinor coupled to gravity in D dimensions, one always ends in the four-dimensional theory with vector-like fermion representations of the gauge group. Applying a theorem due to Atiyah and Hirzebruch[21], it is found that the number of chiral fermions derived from dimensional reduction of a Weyl spinor coupled to Riemannian geometry with $D - 4$ dimensions describing a compact, orientable manifold without boundary is zero. This situation is also found to exist for $N = 8$ supergravity theories. Moreover, except for certain special choices of parameters, the fermions have masses of order the Planck mass. An explanation of the observed small fermion masses requires chiral fermions, where the left-handed and right-handed Weyl or Weyl-Majorana spinors belong to inequivalent representations of the low energy group. Another reason for abandoning the Kaluza-Klein approach to unification is that the gravity theory is not renormalizable. The usual ultraviolet divergences that plague quantum gravity are present, as in the standard point particle four-dimensional quantum gravity theory.

We shall base our unification theory on a D -dimensional Einstein-Yang-Mills-Higgs field theory, with supplementary gauge fields coupled to gravity, which has chiral fermion representations corresponding to massless Dirac modes, and is free of anomalies. We consider an $SO(18) \supset SO(10) \times SO(8)$

model in twelve dimensions in which the eight-dimensional internal space is described by spinor connection gauge fields, associated with an $SO(8)$, which are topologically non-trivial. By dimensional reduction the $SO(18)$ leads to the four-dimensional grand unified theory (GUT) $SO(10)$ and the $SO(8) \supset Sp(4) \times SU(2)$ with three families of chiral quarks and leptons.

The FQFT gauge formalism is applied to the D-dimensional theory to guarantee a self-consistent quantum gravity theory coupled to the Yang-Mills, Higgs and spinor fields. The formalism is free of tachyons and unphysical ghosts and satisfies unitarity to all orders of perturbation theory. It could incorporate supersymmetry if required, in the form of a supergravity theory, but we shall not do so here, in order to aim for as minimal a scheme as possible. The gauge hierarchy problem is resolved because the finite scalar Higgs self-energy loop graphs are damped exponentially at high energies above the physical Higgs scale Λ_H set by the FQFT formalism and by choosing $\Lambda_H \sim 1$ TeV. The compactification scale M_c can be as low as $M_c \simeq 1 - 10$ TeV, while the quantum gravity scale $\Lambda_G \sim 1 - 10$ TeV. This would predict that at these energies future high-energy experiments could detect Kaluza-Klein excitation modes at energies of several TeV.

If we choose $\Lambda_G \sim 1 - 10$ TeV, then quantum gravity loop corrections are perturbatively weak all the way to the Planck energy. This would obviate the need to find a non-perturbative quantum gravity formalism.

2 Kaluza-Klein Theory and The Ground State

We shall begin with the action:

$$W = W_{\text{grav}} + W_{YM} + W_H + W_{\text{Dirac}}, \quad (1)$$

where

$$W_{\text{grav}} = -\frac{1}{\kappa^2} \int d^D z \sqrt{-g} (R + \lambda), \quad (2)$$

$$W_{YM} = -\frac{1}{4} \int d^D z \sqrt{-g} \text{tr}(F^2), \quad (3)$$

$$W_H = -\frac{1}{2} \int d^D z \sqrt{-g} [D_M \phi^a D^M \phi^a + V(\phi^2)], \quad (4)$$

$$W_{\text{Dirac}} = \frac{1}{2} \int d^D z \sqrt{-g} \bar{\psi} \Gamma^A e_A^M [\partial_M \psi - \omega_M \psi - \mathcal{D}(A_M) \psi] + h.c. \quad (5)$$

Here, we use the notation: $z^M = (x^\mu; \mu = 0, 1, 2, 3, y^m; m = 1, 2, \dots, D - 4), M = 0, \dots, D, g = \det(g_{MN})$. The Riemann tensor is defined such that $R_{LM}{}^K{}_N = \partial_L \Gamma_{MN}{}^K - \partial_M \Gamma_{LN}{}^K + \Gamma_{LC}{}^K \Gamma_{MN}{}^C - \Gamma_{MC}{}^K \Gamma_{LN}{}^C$. Moreover, h.c. denotes the Hermitian conjugate, $\bar{\psi} = \psi^\dagger \Gamma^0$, and e_A^M is a vielbein, related to the metric by

$$g_{MN} = \eta_{AB} e_M^A e_N^B, \quad (6)$$

where η_{AB} is the D-dimensional Minkowski metric tensor associated with the flat tangent space with indices A,B,C... Moreover, $F^2 = F_{MN} F^{MN}$, R denotes the scalar curvature, λ is the cosmological constant and

$$F_{aMN} = \partial_N A_{aM} - \partial_M A_{aN} - e f_{abc} A_{bM} A_{cN}, \quad (7)$$

where A_{aM} are the gauge fields of the Yang-Mills group with generators f_{abc} and e is the coupling constant. $\kappa^2 = 16\pi\bar{G}$, where \bar{G} is related to Newton's constant G by $\bar{G} = GV$ and V is the volume of the internal space. The dimensions of \bar{G} are $(\text{length})^\delta (\delta = D - 2)$. Moreover, D_M is the covariant derivative operator:

$$D_M \phi^a = \partial_M \phi^a + e f^{abc} A_M^b \phi^c. \quad (8)$$

The Higgs potential $V(\phi^2)$ is of the form leading to spontaneous symmetry breaking

$$V(\phi^2) = \frac{1}{4} g (\phi^a \phi^a - K^2)^2 + V_0, \quad (9)$$

where V_0 is an adjustable constant and the coupling constant $g > 0$.

The spinor field is minimally coupled to the gauge potential A_M , and \mathcal{D} is a matrix representation of the gauge group G defined in D-dimensions. The spin connection ω_M is

$$\omega_M = \frac{1}{2} \omega_{MAB} \Sigma^{AB}, \quad (10)$$

where $\Sigma^{AB} = \frac{1}{4} [\Gamma^A, \Gamma^B]$ is the spinor matrix associated with the Lorentz algebra $SO(D - 1, 1)$. The components ω_{MAB} satisfy

$$\partial_M e_A^K + \Gamma_{MN}{}^K e_A^N - \omega_{MA}{}^B e_B^K = 0, \quad (11)$$

where $\Gamma_{MN}{}^K$ is the Christoffel symbol.

The field equations for the gravity-Yang-Mills-Higgs-Dirac sector are

$$R_{MN} - \frac{1}{2} g_{MN} R = -\frac{1}{2} \kappa^2 (T_{MN} - \lambda g_{MN}), \quad (12)$$

$$g^{LM}\nabla_L F_{MN} = g^{LM}\left(\partial_L F_{MN} - \Gamma_{LM}^K F_{KN} - \Gamma_{LN}^K F_{MK} + [A_L, F_{MN}]\right) = 0, \quad (13)$$

$$\frac{1}{\sqrt{-g}}D_M[\sqrt{-g}g^{MN}D_N\phi^a] = \left(\frac{\partial V}{\partial\phi^2}\right)\phi^a, \quad (14)$$

$$\Gamma^A e_A^M [\partial_M - \omega_M - \mathcal{D}(A_M)]\psi = 0. \quad (15)$$

The Yang-Mills-Higgs contribution to the energy-momentum tensor is

$$T_{MN}^{\text{YMH}} = \text{tr}(F_{MK}F_N^K) + D_M\phi^a D_N\phi^a - \frac{1}{2}g_{MN}\left[\frac{1}{2}\text{tr}(F^2) + D_P\phi^a D^P\phi^a + V(\phi^2)\right]. \quad (16)$$

We must now choose an ansatz for the ground state of the four-dimensional world. A general theory would start by assuming the ground state to be $M^4 \times B$, where M^4 is four-dimensional Minkowski space and B is a compact internal space. A simple ansatz for the compact space B is to assume a symmetric solution with the structure, $M^4 \times S/H$, where S/H is a coset space of dimension $D - 4$. For the metric we take

$$g_{MN}dz^M dz^N = g_{\mu\nu}(x)dx^\mu dx^\nu + g_{mn}(y)dy^m dy^n, \quad (17)$$

$$A_M^a dz^M = A_\mu^a dy^\mu. \quad (18)$$

One possible symmetric choice for the ground state four-dimensional space-time is that $g_{\mu\nu}$ is a de Sitter solution of the four-dimensional Einstein equation:

$$R_{\mu\nu} = -\frac{1}{2}\kappa^2\lambda g_{\mu\nu}. \quad (19)$$

Since we have additional Yang-Mills and Higgs scalar fields in our higher-dimensional theory, it is possible for us to obtain classical solutions to the field equations in spacetime as the product of flat four-dimensional Minkowski spacetime and an internal compact space. This "spontaneous compactification" can be achieved by going beyond a pure Kaluza-Klein theory, which does not allow a flat four-spacetime unless the curvature of the internal space is also zero[22, 23].

If the metric g_{MN} for the space $M^4 \times B$ is by construction a solution of the Killing equation for $M^4 \times B$, then one can carry out the integrations over

the y coordinates in the action, and the dynamical variables in the theory are functions of x only. The dimensional reduction reduces some of the gauge fields A_M to $A_\mu(x)$, while certain linear combinations of the gauge fields $A_m(x, y)$ become geometrical scalar fields in four dimensions. However, the latter scalar fields do not in general lead to a spontaneous symmetry breaking Higgs mechanism, so that our additional Higgs field action W_H is required to perform this task.

3 Reduction to a Flavor-Chiral Theory

The problem of obtaining the correct quark and lepton quantum numbers is more subtle and difficult than one might suspect in both Kaluza-Klein theories and in string theories. One of the most striking features of particle physics is the knowledge that the quantum numbers of fermions *are not vector-like*, i.e., that left-handed fermions transform under $SU(3) \times SU(2) \times U(1)$ differently from the way right-handed fermions transform. Left-handed quarks are $SU(2)$ doublets but right-handed quarks are $SU(2)$ singlets. Fermions of given helicity form a complex representation of $SU(3) \times SU(2) \times U(1)$, so the fermion representations are not self-conjugate. This fact plays an important role in unified theories, because it means that the bare masses of the quarks and leptons are ruled out by gauge invariance. The fermions can acquire mass only through spontaneous symmetry breaking. Thus, arises the problem of explaining the relative lightness of observed fermions. What generates the smallness of the $SU(2) \times U(1)$ breaking scale?

Since the quantum numbers of the fermions are not vector-like, the spectrum of light fermions depends only on universality class features of an $SU(3) \times SU(2) \times U(1)$ invariant theory, i.e. the lightness of the fermions cannot be modified by any $SU(3) \times SU(2) \times U(1)$ invariant perturbations. We shall consider for the present that the light fermions are massless, ignoring the $SU(2) \times U(1)$ breaking.

Another striking feature of the fermion spectrum is that the anomalous triangle graphs cancel, an important ingredient in a successful gauge invariant unified theory. In addition, each family of fermions consists of five irreducible representations of $SU(3) \times SU(2) \times U(1)$ and there exists a redundancy of three families. The $SU(5)$ and $SO(10)$ grand unified models successfully describe the fermion structure in one family in terms of the representation $\bar{\mathbf{5}}_L + \mathbf{10}_L$ in $SU(5)$, and $\mathbf{16}_L$ in $SO(10)$ [24, 25]. As far as the replication of

families is concerned, it seems natural and elegant to describe this replication in terms of spinor representations of $SO(N)$ for $N \geq 18$.

Witten[26] proved by using topological arguments that, in any number of dimensions, the Dirac operator in a pure Kaluza-Klein theory cannot admit a chiral spectrum. Wetterich[20] showed that the spectrum of fermions could be chiral only if the dimensionality of the space is 2 mod 8. Only if additional Yang-Mills gauge fields are included in a higher-dimensional theory can the fermion spectrum become chiral under dimensional reduction. However, this is only true if the compactification involves a topologically non-trivial configuration of these gauge fields. The addition of extra gauge symmetries is indeed the mechanism whereby superstring theories lead to a spectrum of chiral fermions.

In higher-dimensional theories with a ground state $M^4 \times B$, a massless spectrum of particles is generated as zero modes of wave operators on the internal compact space B . A massless fermion particle in $D = 4 + n$ dimensions obeys

$$\Gamma^M D_M \psi = 0, \quad (20)$$

where Γ^M are the gamma matrices. The quantum numbers remain unchanged in the presence of non-minimal couplings, so we ignore them in the present discussion. We can separate (20) into

$$D^{(4)}\psi + D^{(n)}\psi = 0,$$

where $D^{(4)} = \Gamma^\mu D_\mu$ and $D^{(n)} = \Gamma^m D_m$, and we see that $D^{(n)}$ is a mass operator, whose eigenvalues are the experimentally determined fermion masses. The zero eigenvalues are the massless fermions.

In order to see that a pure Riemannian Kaluza-Klein theory has difficulties, we employ an argument due to Lichnerowicz[27]. If we square the internal Dirac operator, we get $(iD^{(n)})^2 = -D_m D^m + \frac{1}{4}R$, and because $-D^m D_m$ is a non-negative operator, then if $R > 0$ everywhere, it follows that the Dirac operator has no zero eigenvalues, i.e. there are no massless fermions. By using the Atiyah-Hirzebruch theorem[21], which states that the character-valued index of the Dirac operator vanishes on any manifold with a continuous symmetry group (in any even number of dimensions), Witten proved in general that a compact Riemannian manifold (such as the internal space of a pure Kaluza-Klein theory) does not possess zero mass chiral fermions.

We shall now assume that we have additional Yang-Mills gauge fields. If the gauge quantum numbers of the fermions are vector-like, they will re-

main vector-like after compactification, unless special Majorana and Weyl-Majorana conditions are imposed together with gauge symmetry conditions[28]. A serious problem of cancellation of anomalies will occur, unless careful attention is paid to the gauge couplings and the compactification. Non-vector-like couplings of gauge fields will lead to anomalies, unless the condition is satisfied[26]:

$$\text{tr}(M_L^a)^r = \text{tr}(N_R^a)^r, \quad (21)$$

where M_L^a and N_R^a are matrices that couple the gauge fields A_m^a to left-handed and right-handed spin $1/2$ fermions, respectively. Moreover, $r = n+1, n-1, n-3, n-5, \dots$ in $2n$ dimensions with $n+1$ external gluons, $n-1$ external gluons and two gravitons, $n-3$ external gluons and four gravitons, etc. which all correspond to anomalous graphs.

Witten[26] found solutions to Eq.(21) in $2n$ dimensions, which we shall use in the following. Let us consider in $2n$ dimensions, a theory with the orthogonal gauge group $SO(2n+6)$. We assert that the positive chirality spinors of the Lorentz group $SO(2n-1,1)$ transform as positive chirality spinors of the gauge group $SO(2n+6)$, while the negative chirality $SO(2n-1,1)$ spinors transform as negative chirality spinors of $SO(2n+6)$. For any $n > 0$ this leads to an anomaly-free theory, and for $n = 2$, it is the familiar $SO(10)$ grand unified model in four dimensions[25].

We must guarantee that, if we begin with a non-vector-like theory in $2n$ dimensions, then we retain this property under dimensional reduction. The condition

$$\Gamma^{(4+n)} = \Gamma^{(4)} \cdot \Gamma^{(n)}$$

where $\Gamma^{(4+n)} = \Gamma_1 \dots \Gamma_{4+n}$, $\Gamma^4 = \Gamma_1 \dots \Gamma_4$, and $\Gamma^{(n)} = \Gamma_5 \dots \Gamma_{4+n}$, shows that the $4+n$ -dimensional chirality operator $\Gamma^{(4+n)}$ differs from $\Gamma^{(4)}$ by a factor $\Gamma^{(n)}$ that can equal plus 1 or minus 1, so that we can lose the non-vector-like property under compactification. As shown by Randjibar-Daemi, Salam and Strathdee[29], this can be avoided by attributing a non-trivial topological structure to the internal gauge group \mathcal{K} , associated with the internal Kaluza-Klein space. In particular, this can be achieved by inserting a generalized Dirac monopole inside the Kaluza-Klein space.

Let us consider an $SO(18)$ theory in twelve dimensions. We identify the internal space spinor connections ω_j^a with gauge fields B_j^a , which have a non-vanishing vacuum expectation value, $\langle B_j^a \rangle_0 \neq 0$. The B_j^a are $SO(8)$ gauge fields on the eight-dimensional Riemannian manifold of the Kaluza-Klein sector. The $SO(8)$ is embedded in $SO(18)$, such that we obtain the

symmetry breaking $SO(18) \rightarrow SO(8) \times SO(10)$, which leads to the breaking in four dimensions: $SO(8) \times SO(10) \rightarrow SO(10)$. Depending on the type of manifold assumed for the compact dimensions, we get different numbers of families which equal the Euler characteristic of the space B . The number of families in ten dimensions is always even. For example, for the manifolds $S^2 \times S^4$ and CP^3 the number of families of zero mass fermions is four, while for $S^2 \times S^2 \times S^2$ the number of fermion families is eight. For models based on eight and twelve dimensions, the number of fermion families is odd.

Since the observed number of chiral families in four dimensions is three, we shall restrict our attention to the orthogonal group $SO(18)$ in twelve dimensions with an odd number of families. We can decompose the 256-dimensional representation of $SO(18)$ as

$$(\underline{8}_{sp}, \overline{16}) + (\underline{8}'_{sp}, \underline{16})$$

of $SO(8) \times SO(10)$, where $\underline{8}_{sp}$ and $\underline{8}'_{sp}$ are the two real inequivalent spinors of $SO(8)$. Now consider $SO(8) \supset Sp(4) \times SU(2)$, where the vectorial octet $\underline{8}_V$ of $SO(8)$ yields $(\underline{4}, \underline{2})$ of $Sp(4) \times SU(2)$ and $\underline{8}'_{sp}$ of $SO(8)$ equivalently, whereas $\underline{8}_{sp}$ of $SO(8)$ yields

$$(\underline{1}, \underline{3}) + (\underline{5}, \underline{1})$$

of $Sp(4) \times SU(2)$. This then gives for the 256 representation of $SO(18)$:

$$(\underline{1}, \underline{3}, \underline{16}) + (\underline{5}, \underline{1}, \underline{16}) + (\underline{4}, \underline{2}, \overline{16})$$

of $Sp(4) \times SU(2) \times SO(10)$. By identifying $Sp(4)$ as a supplementary factor of the exactly conserved colour group $SU^c(3) \times Sp^c(4)'$, and $SU(2)$ as a gauged family subgroup of $SO(18)$, then only the fundamental left-handed fermions without primed colour are three families of the sixteen-dimensional representation of $SO(10)$, which agrees with observations[30, 31].

4 Finite Quantum Field Theory Formalism in D Dimensions

It is well-known that higher-dimensional field theories are non-renormalizable, for their structure has enhanced divergences. The non-renormalizability arises from the presence of infinite towers of non-chiral Kaluza-Klein states

which circulate in all Feynman graphs. Even if we choose to describe the space as consisting of D-dimensional flat spacetime, the field theory would still be non-renormalizable, because we need to integrate over D dimensions of uncompactified loop momenta. Therefore, the well-known non-renormalizability of quantum gravity is further exacerbated in higher-dimensional theories. However, by applying our finite quantum field theory (FQFT) formalism, based on a nonlocal interaction Lagrangian which is perturbatively finite, unitary and gauge invariant [10-18], we can obtain a finite quantum field theory in higher dimensions and, in contrast to string theory, achieve a *genuine quantum field theory*, which allows vertex operators to be taken off the mass shell. The finiteness draws from the fact that factors of $\exp[\mathcal{K}(p^2)/2\Lambda^2]$ are attached to propagators which suppress any ultraviolet divergences in Euclidean momentum space, where Λ is an energy scale factor. An important feature of FQFT is *that only the quantum loop graphs have non-local properties*; the classical tree graph theory retains full causal and local behaviour.

An important development in FQFT was the discovery that gauge invariance and unitarity can be restored by adding series of higher interactions. The resulting theory possesses a nonlinear, field representation dependent gauge invariance which agrees with the original local symmetry on shell but is larger off shell. Quantization is performed in the functional formalism using an analytic and convergent measure factor which retains invariance under the new symmetry. An explicit calculation was made of the measure factor in QED[11], and it was obtained to lowest order in Yang-Mills theory[14]. Kleppe and Woodard[16] obtained an ansatz based on the derived dimensionally regulated result when $\Lambda \rightarrow \infty$, which was conjectured to lead to a general functional measure factor in FQFT gauge theories.

A convenient formalism which makes the FQFT construction transparent is based on shadow fields[14, 16]. We shall consider the D-dimensional spacetime to be approximately flat Minkowski spacetime, which is a valid approximation for circles in the internal space B having large fixed radii R .

Let us denote by f_i a generic local field and write the standard local action as

$$W[f] = W_F[f] + W_I[f], \quad (22)$$

where W_F and W_I denote the free part and the interaction part of the action, respectively, and

$$W_F = \frac{1}{2} \int d^D z f_i \mathcal{K}_{ij} f_j. \quad (23)$$

In a gauge theory W would be the Becchi, Rouet, Stora, Tyutin (BRST) gauge-fixed action including ghost fields in the invariant action required to fix the gauge[32]. The kinetic operator \mathcal{K} is fixed by defining a Lorentz-invariant distribution operator:

$$\mathcal{E} \equiv \exp\left(\frac{\mathcal{K}}{2\Lambda^2}\right) \quad (24)$$

and the shadow operator:

$$\mathcal{O}^{-1} = \frac{\mathcal{K}}{\mathcal{E}^2 - 1}. \quad (25)$$

Every local field f_i has an auxiliary counterpart field h_i , and they are used to form a new action:

$$W[f, h] \equiv W_F[\hat{f}] - P[h] + W_I[f + h], \quad (26)$$

where

$$\hat{f} = \mathcal{E}^{-1}f, \quad P[h] = \frac{1}{2} \int d^D z h_i \mathcal{O}_{ij}^{-1} h_j.$$

By iterating the equation

$$h_i = \mathcal{O}_{ij} \frac{\delta W_I[f + h]}{\delta h_j} \quad (27)$$

the shadow fields can be determined as functions, and the regulated action is derived from

$$\hat{W}[f] = W[f, h(f)]. \quad (28)$$

We recover the original local action when we take the limit $\Lambda \rightarrow \infty$ and $\hat{f} \rightarrow f, h(f) \rightarrow 0$.

Quantization is performed using the definition

$$\langle 0 | T^*(O[f]) | 0 \rangle_{\mathcal{E}} = \int [Df] \mu[f] (\text{gauge fixing}) O[\hat{f}] \exp(i\hat{W}[f]). \quad (29)$$

On the left-hand side we have the regulated vacuum expectation value of the T^* -ordered product of an arbitrary operator $O[f]$ formed from the local fields f_i . The subscript \mathcal{E} signifies that a regulating Lorentz distribution has been used. Moreover, $\mu[f]$ is a measure factor and there is a gauge fixing factor, both of which are needed to maintain perturbative unitarity in gauge theories.

The new Feynman rules for FQFT are obtained as follows: The vertices remain unchanged but every leg of a diagram is connected either to a regularized propagator,

$$\frac{i\mathcal{E}^2}{\mathcal{K} + i\epsilon} = -i \int_1^\infty \frac{d\tau}{\Lambda^2} \exp\left(\tau \frac{\mathcal{K}}{\Lambda^2}\right), \quad (30)$$

or to a shadow propagator,

$$-i\mathcal{O} = \frac{i(1 - \mathcal{E}^2)}{\mathcal{K}} = -i \int_0^1 \frac{d\tau}{\Lambda^2} \exp\left(\tau \frac{\mathcal{K}}{\Lambda^2}\right). \quad (31)$$

The formalism is set up in Minkowski spacetime and loop integrals are formally defined in Euclidean space by performing a Wick rotation. This facilitates the analytic continuation; the whole formalism could from the outset be developed in Euclidean space.

In FQFT renormalization is carried out as in any other field theory. The bare parameters are calculated from the renormalized ones and Λ , such that the limit $\Lambda \rightarrow \infty$ is finite for all noncoincident Green's functions, and the bare parameters are those of the local theory. The regularizing interactions *are determined by the local operators*.

The regulating Lorentz distribution function \mathcal{E} must be chosen to perform an explicit calculation in perturbation theory. We do not know the unique choice of \mathcal{E} [33]. It maybe that there exists an equivalence mapping between all the possible distribution functions \mathcal{E} . However, once a choice for the function is made, then the theory and the perturbative calculations are uniquely fixed. A standard choice in early FQFT papers is[10, 11]:

$$\mathcal{E}_m = \exp\left(\frac{\partial^2 - m^2}{2\Lambda^2}\right). \quad (32)$$

An explicit construction for QED was given using the Cutkosky rules as applied to FQFT whose propagators have poles only where $\mathcal{K} = 0$ and whose vertices are entire functions of \mathcal{K} . The regulated action $\hat{W}[f]$ satisfies these requirements which guarantees unitarity on the physical space of states. The local action is gauge fixed and then a regularization is performed on the BRST theory.

The infinitesimal transformation

$$\delta f_i = T_i(f) \quad (33)$$

generates a symmetry of W , and the infinitesimal transformation

$$\hat{\delta}f_i = \mathcal{E}_{ij}^2 T_j(f + h[f]) \quad (34)$$

generates a symmetry of the regulated action \hat{W} . It follows that FQFT regularization preserves all continuous symmetries including supersymmetry. The quantum theory will preserve symmetries provided a suitable measure factor can be found such that

$$\hat{\delta}([Df]\mu[f]) = 0. \quad (35)$$

Moreover, the interaction vertices of the measure factor must be entire functions of the operator \mathcal{K} and they must not destroy the FQFT finiteness.

In FQFT tree order, Green's functions remain local except for external lines which are unity on shell. It follows immediately that since on-shell tree amplitudes are unchanged by the regularization, \hat{W} preserves all symmetries of W on shell. Also all loops contain at least one regularizing propagator and therefore are ultraviolet finite. Shadow fields are eliminated at the classical level, for functionally integrating over them would produce divergences from shadow loops. Since shadow field propagators do not contain any poles there is no need to quantize the shadow fields. Feynman rules for $\hat{W}[f, h]$ are as simple as those for local field theory.

5 Finite Quantum Yang-Mills and Kaluza-Klein Gravity Theory

Let us now consider the finite quantization of the D-dimensional Yang-Mills sector in D-dimensional Minkowski flat space. The gauge field strength F_{aMN} is invariant under the familiar transformations:

$$\delta A_{aM} = -\partial_M \theta_a + e f_{abc} A_{bM} \theta_c.$$

To regularize the Yang-Mills sector, we identify the kinetic operator

$$\mathcal{K}_{ab}^{MN} = \delta_{ab}(\partial^2 \eta^{MN} - \partial^M \partial^N).$$

The regularized action is given by[14]

$$\hat{W}_{YM}[A] = \frac{1}{2} \int d^D x \left\{ \hat{A}_{aM} \mathcal{K}_{ab}^{MN} \hat{A}_{bN} - B_{aM}[A] (\mathcal{O}_{ab}^{MN})^{-1} B_{bN}[A] \right\}$$

$$+W_{YM}^I[A+B[A]], \quad (36)$$

where B_{aM} is the Yang-Mills shadow field, which satisfies the expansion:

$$\begin{aligned} B_a^M[A] &= \mathcal{O}_{ab}^{MN} \frac{\delta W_{YM}^I[A+B]}{\delta B_b^N} \\ &= \mathcal{O}_{ab}^{MN} e f_{bcd} [A_{Nc} \partial_K A_d^K + A_{cK} \partial_N A_d^K - 2A_{cK} \partial^K A_d^N] + O(e^2 A^3). \end{aligned}$$

The regularized gauge symmetry transformation is

$$\begin{aligned} \hat{\delta}_\theta A_a^M &= (\mathcal{E}_{ab}^{MN})^2 \left\{ -\partial^M \theta_a + e f_{bcd} (A_{cN} + B_{cN}[A]) \theta_d \right\} \\ &\quad - \partial^M \theta_a + (\mathcal{E}_{ab}^{MN})^2 e f_{bcd} (A_{cN} + B_{cN}[A]) \theta_d. \end{aligned}$$

The extended gauge transformation is neither linear nor local.

We functionally quantize the Yang-Mills sector using

$$\langle 0|T^*(O[A])|0\rangle_{\mathcal{E}} = \int [DA] \mu[A] (\text{gauge fixing}) O[\hat{A}] \exp(i\hat{W}_{YM}[A]). \quad (37)$$

To fix the gauge we use Becchi-Rouet-Stora-Tyutin (BRST)[32] invariance. The ghost structure of the BRST action comes from exponentiating the Faddeev-Popov determinant. Since the FQFT algebra fails to close off-shell, we need to introduce higher ghost terms into both the action and the BRST transformation. In Feynman gauge, the local BRST Lagrangian is

$$\begin{aligned} \mathcal{L}_{YM\ BRST} &= -\frac{1}{2} \partial_M A_{aN} \partial^M A_a^N - \partial^M \bar{\eta}_a \partial_M \eta_a + e f_{abc} \partial^M \bar{\eta}_a A_{bM} \eta_c \\ &\quad + e f_{abc} \partial_M A_{aN} A_b^M A_c^N - \frac{1}{4} e^2 f_{abc} f_{cde} A_{aM} A_{bN} A_d^M A_c^N. \end{aligned}$$

It is invariant under the global symmetry transformation:

$$\begin{aligned} \delta A_{aM} &= (\partial_M \eta_a - e f_{abc} A_{bM} \eta_c) \delta \zeta, \\ \delta \eta_a &= -\frac{1}{2} e f_{abc} \eta_b \eta_c \delta \zeta, \\ \delta \bar{\eta}_a &= -\partial_M A_a^M \delta \zeta, \end{aligned}$$

where ζ is a constant anticommuting c-number.

The gluon and ghost kinetic operators are

$$\mathcal{K}_{ab}^{MN} = \delta_{ab} \eta^{MN} \partial^2,$$

$$\mathcal{K}_{ab} = \delta_{ab} \partial^2.$$

The regularizing operators associated with the ghosts are

$$\bar{\mathcal{E}} = \exp\left(\frac{\partial^2}{2\Lambda^2}\right),$$

$$\bar{\mathcal{O}} = \frac{\bar{\mathcal{E}}^2 - 1}{\partial^2}.$$

The regularized BRST action is

$$\begin{aligned} \hat{W}_{YM}[A, \bar{\eta}, \eta] = \int d^D z \Big\{ & -\frac{1}{2} \partial_N \hat{A}_{aM} \partial^N \hat{A}_a^M - \frac{1}{2} B_{aM} \bar{\mathcal{O}}^{-1} B_a^M \\ & - \partial^M \hat{\eta}_a \partial_M \hat{\eta}_a - \bar{\chi}_a \bar{\mathcal{O}}^{-1} \chi_a \Big\} + W_{YM}^I[A + B, \bar{\eta} + \bar{\chi}, \eta + \chi], \end{aligned}$$

where χ is the ghost shadow field.

The regularizing, nonlocal BRST symmetry transformation is

$$\hat{\delta} A_{aM} = \bar{\mathcal{E}}^2 \left\{ (\partial_M \eta_a + \partial_M \chi_a) - e f_{abc} (A_{bM} + B_{bM}) (\eta_c + \chi_c) \right\} \delta \zeta,$$

$$\hat{\delta} \eta_a = -\frac{1}{2} e f_{abc} \bar{\mathcal{E}}^2 (\eta_b + \chi_b) (\eta_c + \chi_c) \delta \zeta,$$

$$\hat{\delta} \bar{\eta}_a = -\bar{\mathcal{E}}^2 (\partial_M A_a^M + \partial_M B_a^M) \delta \zeta.$$

The full functional, gauge fixed quantization is now given by

$$\begin{aligned} \langle 0 | T^*(O[A, \bar{\eta}, \eta]) | 0 \rangle_{\mathcal{E}} &= \int [DA][D\bar{\eta}][D\eta] \mu[A, \bar{\eta}, \eta] O[\hat{A}, \hat{\bar{\eta}}, \hat{\eta}] \\ &\quad \exp(i \hat{W}_{YM}[A, \bar{\eta}, \eta]). \end{aligned} \tag{38}$$

Kleppe and Woodard[14] have obtained the invariant measure factor for the regularized Yang-Mills sector to first order in the coupling constant e :

$$\ln(\mu[A, \bar{\eta}, \eta]) = -\frac{1}{2} e^2 f_{acd} f_{bcd} \int d^D z A_{aM} \mathcal{M} A_b^M + O(e^3), \tag{39}$$

where

$$\mathcal{M} = \frac{1}{2^D \pi^{D/2}} \int_0^1 d\tau \frac{\Lambda^{D-2}}{(\tau+1)^{D/2}} \exp\left(\frac{\tau}{\tau+1} \frac{\partial^2}{\Lambda^2}\right) \left\{ \frac{2}{\tau+1} + 1 - D + 2(D-1) \frac{\tau}{\tau+1} \right\}.$$

The existence of a suitable invariant measure factor implies that the necessary Slavnov-Taylor identities also exist.

We shall now formulate in more detail the Kaluza-Klein gravitational sector as a FQFT. This problem has been considered previously in the context of four-dimensional GR[10, 11, 18]. We shall treat the theory as effectively being in D flat Minkowski dimensions, $D = 4 + d$. A spacetime consisting of four flat Minkowski dimensions and d circles of fixed radii $R = 1/\mu_0$ cannot, in general, be equivalent to a flat D -dimensional spacetime. However, as the energy scale μ increases, the effective length scale decreases, so that the fixed radius R appears to become large, and the D -dimensional flat spacetime becomes a good approximation. In fact, FQFT can be formulated as a perturbative theory by expanding around any fixed, classical metric background, but for the sake of simplicity, we shall only consider in the following expansions about flat spacetime. As in ref.([18]), we will regularize the GR equations using the shadow field formalism.

We expand the local interpolating field $\mathbf{g}^{MN} = \sqrt{-g}g^{MN}$ ($g = \text{Det}(g_{MN})$) about Minkowski spacetime

$$\mathbf{g}^{MN} = \eta^{MN} + \kappa \gamma^{MN} + O(\kappa^2). \quad (40)$$

We separate the free and interacting parts of the action

$$W_{\text{grav}}(g) = W_{\text{grav}}^F(g) + W_{\text{grav}}^I(g). \quad (41)$$

The finite regularized gravitational action in FQFT is given by

$$\hat{W}_{\text{grav}}(g, s) = W_{\text{grav}}^F(\hat{g}) - P_{\text{grav}}(s) + W_{\text{grav}}^I(g + s), \quad (42)$$

where

$$\hat{g} = \mathcal{E}^{-1}g, \quad P_{\text{grav}}(s) = \frac{1}{2} \int d^D z \mathcal{F}(\sqrt{s}, s_i \mathcal{O}_{ij}^{-1} s_j), \quad (43)$$

s denotes the graviton shadow field, and \mathcal{F} denotes the detailed expansion of the metric tensor formed from the shadow field.

The quantum gravity perturbation theory is locally $SO(D-1, 1)$ invariant (generalized, nonlinear field representation dependent transformations), unitary and finite to all orders in a way similar to the non-Abelian gauge theories formulated using FQFT. At the tree graph level all unphysical polarization states are decoupled and nonlocal effects will only occur in graviton and graviton-matter loop graphs. Because the gravitational tree graphs are purely local there is a well-defined classical GR limit. The finite quantum gravity theory is well-defined in D real spacetime dimensions.

The graviton regularized propagator in a fixed de Donder gauge is given in D -dimensional Minkowski space by [34, 9]

$$D_{MNKL}^{\text{grav}} = (\eta_{MK}\eta_{NL} + \eta_{ML}\eta_{NK} - \eta_{MN}\eta_{KL}) \\ \left(\frac{-i}{(2\pi)^D} \right) \int d^D k \frac{\mathcal{E}^2(k^2)}{k^2 - i\epsilon} \exp[ik \cdot (z - z')],$$

while the shadow propagator is

$$D_{MNKL}^{\text{shad}} = (\eta_{MK}\eta_{NL} + \eta_{ML}\eta_{NK} - \eta_{MN}\eta_{KL}) \\ \left(\frac{-i}{(2\pi)^D} \right) \int d^D k \frac{[1 - \mathcal{E}^2(k^2)]}{k^2 - i\epsilon} \exp[ik \cdot (z - z')].$$

In momentum space we have

$$\frac{-i\mathcal{E}^2(k^2)}{k^2 - i\epsilon} = -i \int_1^\infty \frac{d\tau}{\Lambda_G^2} \exp\left(-\tau \frac{k^2}{\Lambda_G^2}\right),$$

and

$$\frac{i(\mathcal{E}^2(k^2) - 1)}{k^2 - i\epsilon} = -i \int_0^1 \frac{d\tau}{\Lambda_G^2} \exp\left(-\tau \frac{k^2}{\Lambda_G^2}\right),$$

where Λ_G is the gravitational scale parameter.

We quantize by means of the path integral operation

$$\langle 0|T^*(O[g])|0\rangle_\mathcal{E} = \int [Dg] \mu[g] (\text{gauge fixing}) O[\hat{g}] \exp(i\hat{W}_{\text{grav}}[g]). \quad (44)$$

The quantization is carried out in the functional formalism by finding a measure factor $\mu[g]$ to make $[Dg]$ invariant under the classical symmetry. To ensure a correct gauge fixing scheme, we write $W_{\text{grav}}[g]$ in the BRST invariant form with ghost fields; the ghost structure arises from exponentiating

the Faddeev-Popov determinant. The algebra of gauge symmetries is not expected to close off-shell, so one needs to introduce higher ghost terms (beyond the normal ones) into both the action and the BRST transformation. The BRST action will be regularized directly to ensure that all the corrections to the measure factor are included.

6 Quantum Nonlocal Behavior in FQFT

In FQFT, it can be argued that the extended objects that replace point particles (the latter are obtained in the limit $\Lambda \rightarrow \infty$) cannot be probed because of a Heisenberg uncertainty type of argument. The FQFT nonlocality *only occurs at the quantum loop level*, so there is no noncausal classical behavior. In FQFT the strength of a signal propagated over an invariant interval l^2 outside the light cone would be suppressed by a factor $\exp(-l^2\Lambda^2)$.

Nonlocal field theories can possess non-perturbative instabilities. These instabilities arise because of extra canonical degrees of freedom associated with higher time derivatives. If a Lagrangian contains up to N time derivatives, then the associated Hamiltonian is linear in $N - 1$ of the corresponding canonical variables and extra canonical degrees of freedom will be generated by the higher time derivatives. The nonlocal theory can be viewed as the limit $N \rightarrow \infty$ of an N th derivative Lagrangian. Unless the dependence on the extra solutions is arbitrarily choppy in the limit, then the higher derivative limit will produce instabilities[35]. The condition for the smoothness of the extra solutions is that no invertible field redefinition exists which maps the nonlocal field equations into the local ones. String theory does satisfy this smoothness condition as can be seen by inspection of the S-matrix tree graphs. In FQFT the tree amplitudes agree with those of the local theory, so the smoothness condition is not obeyed.

It was proved by Kleppe and Woodard[14] that the solutions of the non-local field equations in FQFT are in one-to-one correspondence with those of the original local theory. The relation for a generic field v_i is

$$v_i^{\text{nonlocal}} = \mathcal{E}_{ij}^2 v_j^{\text{local}}. \quad (45)$$

Also the actions satisfy

$$W[v] = \hat{W}[\mathcal{E}^2 v]. \quad (46)$$

Thus, there are no extra classical solutions. The solutions of the regularized nonlocal Euler-Lagrange equations are in one-to-one correspondence with

those of the local action. It follows *that the regularized nonlocal FQFT is free of higher derivative solutions, so FQFT can be a stable theory.*

Since only the quantum loop graphs in the nonlocal FQFT differ from the local field theory, then FQFT can be viewed as a non-canonical quantization of fields which obey the local equations of motion. Provided the functional quantization in FQFT is successful, then the theory does maintain perturbative unitarity.

7 A Resolution of The Higgs Hierarchy Problem and Quantum Gravity

It is time to discuss the Higgs sector hierarchy problem[36]. The gauge hierarchy problem is related to the spin 0^+ scalar field nature of the Higgs particle in the standard model with quadratic mass divergence and no protective extra symmetry at $m = 0$. In standard point particle, local field theory the fermion masses are logarithmically divergent and there exists a chiral symmetry restoration at $m = 0$.

Writing $m_H^2 = m_{0H}^2 + \delta m_H^2$, where m_{0H} is the bare Higgs mass and δm_H is the Higgs self-energy renormalization constant, we get for the one loop Feynman graph in $D = 4$ spacetime:

$$\delta m_H^2 \sim \frac{g}{32\pi^2} \Lambda_C^2, \quad (47)$$

where Λ_C is a cutoff parameter. If we want to understand the nature of the Higgs mass we must require that

$$\delta m_H^2 \leq O(m_H^2), \quad (48)$$

i.e. the quadratic divergence should be cut off at the mass scale of the order of the physical Higgs mass. Since $m_H \simeq \sqrt{2g}v$, where $v = \langle \phi \rangle_0$ is the vacuum expectation value of the scalar field ϕ and $v = 246$ GeV from the electroweak theory, then in order to keep perturbation theory valid, we must demand that $10 \text{ GeV} \leq m_H \leq 350 \text{ GeV}$ and we need

$$\Lambda_C = \Lambda_{\text{Higgs}} \leq 1 \text{ TeV}, \quad (49)$$

where the lower bound on m_H comes from the avoidance of washing out the spontaneous symmetry breaking of the vacuum.

Nothing in the standard model can tell us why (49) should be true, so we must go beyond the standard model to solve the problem. Λ_C is an arbitrary parameter in point particle field theory with no physical interpretation. Since all particles interact through gravity, then ultimately we should expect to include gravity in the standard model, so we expect that $\Lambda_{\text{Planck}} \sim 10^{19}$ GeV should be the natural cutoff. Then we have using (49) and $g \sim 1$:

$$\frac{\delta m_H^2(\Lambda_{\text{Higgs}})}{\delta m_H^2(\Lambda_{\text{Planck}})} \approx \frac{\Lambda_{\text{Higgs}}^2}{\Lambda_{\text{Planck}}^2} \approx 10^{-34},$$

which represents an intolerable fine-tuning of parameters. This ‘naturalness’ or hierarchy problem is one of the most serious defects of the standard model.

There have been two strategies proposed as ways out of the hierarchy problem. The Higgs is taken to be composite at a scale $\Lambda_C \simeq 1$ TeV, thereby providing a natural cutoff in the quadratically divergent Higgs loops. One such scenario is the ‘technicolor’ model, but it cannot be reconciled with the accurate standard model data, nor with the smallness of fermion masses and the flavor-changing neutral current interactions. The other strategy is to postulate supersymmetry, so that the opposite signs of the boson and fermion lines cancel by means of the non-renormalization theorem. However, supersymmetry is badly broken at lower energies, so we require that

$$\delta m_H^2 \sim \frac{g}{32\pi^2} |\Lambda_C^2 \text{ bosons} - \Lambda_C^2 \text{ fermions}| \leq 1 \text{ TeV}^2,$$

or, in effect

$$|m_B - m_F| \leq 1 \text{ TeV}.$$

This physical requirement leads to the prediction that the supersymmetric partners of known particles should have a threshold ≤ 1 TeV.

A third possible strategy is to introduce a FQFT formalism, and realize a field theory mechanism which will introduce a natural physical scale in the theory ≤ 1 TeV, which will protect the Higgs mass from becoming large and unstable.

Let us consider the regularized scalar field FQFT Lagrangian in D-dimensional Minkowski space

$$\hat{\mathcal{L}}_S = \frac{1}{2} \hat{\phi}(\partial^2 - m^2)\hat{\phi} - \frac{1}{2} \rho \mathcal{O}^{-1} \rho + \frac{1}{2} Z^{-1} \delta m^2 (\phi + \rho)^2 - \frac{1}{24} g_0 (\phi + \rho)^4, \quad (50)$$

where $\phi = Z^{1/2} \phi_R$ is the bare field, ϕ_R is the renormalized field, $\hat{\phi} = \mathcal{E}^{-1} \phi$, ρ is the shadow field, m_0 is the bare mass, Z is the field strength renormalization

constant, δm^2 is the mass renormalization constant and m is the physical mass. The regularizing operator is given by

$$\mathcal{E}_m = \exp\left(\frac{\partial^2 - m^2}{2\Lambda_H^2}\right), \quad (51)$$

while the shadow kinetic operator is

$$\mathcal{O}^{-1} = \frac{\partial^2 - m^2}{\mathcal{E}_m^2 - 1}. \quad (52)$$

Here, Λ_H is the Higgs scalar field energy scale in FQFT.

The full propagator is

$$-i\Delta_R(p^2) = \frac{-i\mathcal{E}_m^2}{p^2 + m^2 - i\epsilon} = -i \int_1^\infty \frac{d\tau}{\Lambda_H^2} \exp\left[-\tau\left(\frac{p^2 + m^2}{\Lambda_H^2}\right)\right], \quad (53)$$

whereas the shadow propagator is

$$i\Delta_{\text{shadow}} = i \frac{\mathcal{E}_m^2 - 1}{p^2 + m^2} = -i \int_0^1 \frac{d\tau}{\Lambda_H^2} \exp\left[-\tau\left(\frac{p^2 + m^2}{\Lambda_H^2}\right)\right]. \quad (54)$$

Let us define the self-energy $\Sigma(p)$ as a Taylor series expansion around the mass shell $p^2 = -m^2$:

$$\Sigma(p^2) = \Sigma(-m^2) + (p^2 + m^2) \frac{\partial \Sigma}{\partial p^2}(-m^2) + \tilde{\Sigma}(p^2), \quad (55)$$

where $\tilde{\Sigma}(p^2)$ is the usual finite part in the point particle limit $\Lambda_H \rightarrow \infty$. We have

$$\tilde{\Sigma}(-m^2) = 0, \quad (56)$$

and

$$\frac{\partial \tilde{\Sigma}(p^2)}{\partial p^2}(p^2 = -m^2) = 0. \quad (57)$$

The full propagator is related to the self-energy $\Sigma(p^2)$ by

$$-i\Delta_R(p^2) = \frac{-i\mathcal{E}_m^2[1 + \mathcal{O}\Sigma(p^2)]}{p^2 + m^2 + \Sigma(p^2)} = \frac{-iZ}{p^2 + m^2 + \Sigma_R(p^2)}. \quad (58)$$

Here $\Sigma_R(p^2)$ is the renormalized self-energy which can be written as

$$\Sigma_R(p^2) = (p^2 + m^2) \left[\frac{Z}{\mathcal{E}_m^2(1 + \mathcal{O}\Sigma)} - 1 \right] + \frac{Z\Sigma}{\mathcal{E}_m^2(1 + \mathcal{O}\Sigma)}. \quad (59)$$

The 1PI two-point function is given by

$$-i\Gamma_R^{(2)}(p^2) = i[\Delta_R(p^2)]^{-1} = \frac{i[p^2 + m^2 + \Sigma(p^2)]}{\mathcal{E}_m^2[1 + \mathcal{O}\Sigma(p^2)]}. \quad (60)$$

Since $\mathcal{E}_m \rightarrow 1$ and $\mathcal{O} \rightarrow 0$ as $\Lambda_H \rightarrow \infty$, then in this limit

$$-i\Gamma_R^{(2)}(p^2) = i[p^2 + m^2 + \Sigma(p^2)], \quad (61)$$

which is the standard point particle result.

The mass renormalization is determined by the propagator pole at $p^2 = -m^2$ and we have

$$\Sigma_R(-m^2) = 0. \quad (62)$$

Also, we have the condition

$$\frac{\partial \Sigma_R(p^2)}{\partial p^2}(p^2 = -m^2) = 0. \quad (63)$$

The renormalized coupling constant is defined by the four-point function $\Gamma_R^{(4)}(p_1, p_2, p_3, p_4)$ at the point $p_i = 0$:

$$\Gamma_R^{(4)}(0, 0, 0, 0) = g. \quad (64)$$

The bare coupling constant g_0 is determined by

$$Z^2 g_0 = g + \delta g(g, m^2, \Lambda_H^2). \quad (65)$$

Moreover,

$$\begin{aligned} Z &= 1 + \delta Z(g, m^2, \Lambda_H^2), \\ Zm_0^2 &= Zm^2 - \delta m^2(g, m^2, \Lambda_H^2). \end{aligned}$$

A calculation of the scalar field mass renormalization gives[16]:

$$\delta m^2 = \frac{g}{2^{D+1}\pi^{D/2}} m^{D-2} \Gamma\left(1 - \frac{D}{2}, \frac{m^2}{\Lambda_H^2}\right) + O(g^2), \quad (66)$$

where $\Gamma(n, z)$ is the incomplete gamma function:

$$\Gamma(n, z) = \int_z^\infty \frac{dt}{t} t^n \exp(-t) = (n-1)\Gamma(n-1, z) + z^{n-1} \exp(-z). \quad (67)$$

We have

$$\Gamma(-1, z) = -E_i(z) + \frac{1}{z} \exp(-z), \quad (68)$$

where $E_i(z)$ is the exponential integral

$$E_i(z) \equiv \int_z^\infty dt \frac{\exp(-t)}{t}.$$

For small z we obtain the expansion

$$E_i(z) = -\ln(z) - \gamma + z - \frac{z^2}{2 \cdot 2!} + \frac{z^3}{3 \cdot 3!} - \dots, \quad (69)$$

where γ is Euler's constant. For large positive values of z , we have the asymptotic expansion

$$E_i(z) \sim \exp(-z) \left[\frac{1}{z} - \frac{1}{z^2} + \frac{2!}{z^3} - \dots \right]. \quad (70)$$

Thus, for small m/Λ_H we obtain in $D = 4$ spacetime:

$$\delta m^2 = \frac{g}{32\pi^2} \left[\Lambda_H^2 - m^2 \ln\left(\frac{\Lambda_H^2}{m^2}\right) - m^2(1 - \gamma) + O\left(\frac{m^2}{\Lambda_H^2}\right) \right] + O(g^2), \quad (71)$$

which is the standard quadratically divergent self-energy, obtained from a cutoff procedure or a dimensional regularization scheme.

We have for $z \rightarrow \infty$:

$$\Gamma(a, z) \sim z^{a-1} \exp(-z) \left[1 + \frac{a-1}{z} + O\left(\frac{1}{z^2}\right) \right] \quad (72)$$

so that for $m \gg \Lambda_H$, we get in D-dimensional space

$$\delta m^2 \sim \frac{g}{2^{D+1}\pi^{D/2}} \left(\frac{\Lambda_H^D}{m^2}\right) \exp\left(-\frac{m^2}{\Lambda_H^2}\right). \quad (73)$$

Thus, the Higgs self-energy one loop graph falls off exponentially fast for $m \gg \Lambda_H$.

The lowest order contributions to the graviton self-energy in FQFT will include the standard graviton loops, the shadow field graviton loops, the ghost field loop contributions with their shadow field counterparts, and the measure loop contributions. The calculated measures for regularized QED,

first order Yang-Mills theory and all orders in ϕ^4 and ϕ^6 theories lead to self-energy contributions that are controlled by an incomplete Γ function. For the regularized perturbative gravity theory the first order loop amplitude is

$$A_i = \Gamma\left(2 - D/2, \frac{p^2}{\Lambda_G^2}\right) (p^2)^{D-2} F_i(D/2). \quad (i = 1, \dots, 5) \quad (74)$$

The dimensional regularization result is obtained by the replacement

$$\Gamma\left(2 - D/2, \frac{p^2}{\Lambda_G^2}\right) \rightarrow \Gamma(2 - D/2),$$

yielding the result[37, 38]

$$A_i \sim \Gamma(2 - D/2) (p^2)^{D-2} F_i(D/2) \sim \frac{1}{\epsilon} (p^2)^{D-2} F_i(D/2), \quad (75)$$

where $\epsilon = 2 - D/2$ and $\Gamma(n)$ is the gamma function. Whereas the dimensional regularization result is singular in the limit $\epsilon \rightarrow 0$, the FQFT result is *finite in this limit* for a fixed value of the parameter Λ_G , resulting in a finite graviton self-energy amplitude A_i .

For D-dimensional spacetime, using (72) and (74), we obtain in the Euclidean momentum limit

$$A_i \sim \left(\frac{p^2}{\Lambda_G^2}\right)^{1-D/2} \exp\left(-\frac{p^2}{\Lambda_G^2}\right) (p^2)^{D-2} F_i(D/2) \quad (i = 1, \dots, 5). \quad (76)$$

Thus, in the infinite Euclidean momentum limit the quantum graviton self-energy contribution damps out and quantum graviton corrections become negligible. It is often argued in the literature on quantum gravity that the gravitational quantum corrections scale as $\alpha_G = GE^2$, so that for sufficiently large values of the energy E , namely, of order the Planck energy, the gravitational quantum fluctuations become large. We see that in FQFT quantum gravity this may not be the case, because for $\Lambda_G \ll M_{\text{Planck}} \sim 10^{19}$ GeV, the finite quantum loop corrections become negligible in the high energy limit. Of course, the contributions of the tree graph exchanges of virtual gravitons can be large in the high energy limit, corresponding to strong classical gravitational fields. It follows that for high enough energies, a classical curved spacetime would be a good approximation.

In contrast to recent models of branes and strings in which the compactification scale is lowered to the TeV range[4, 8], we retain the classical GR

gravitation picture and its Newtonian limit. It is perhaps a radical notion to entertain that quantum gravity becomes weaker as the energy scale increases towards the Planck scale $\sim 10^{19}$ GeV, but there is, of course, no known experimental reason why this should not be the case in nature. This would have important implications about the nature of singularities in gravitational collapse and the big bang scenario, for we could not appeal to quantum gravity to alleviate the singularity problem, but hope instead that a classical modification of GR occurs for very small distances $\leq 10^{-33}$ cm.

If we choose $\Lambda_H = \Lambda_G \geq 1$ -5 TeV, then due to the damping of the gravitational loop graphs and the scalar loop graphs in the Euclidean limit $p^2 \gg \Lambda^2$, the Higgs sector is protected from large unstable radiative corrections and FQFT provides a solution to the Higgs hierarchy problem, without invoking low-energy supersymmetry or technicolor. The universal fixed FQFT scale Λ_H corresponds to the fundamental length $\ell_H \leq 10^{-17}$ cm. For a Higgs mass much larger than 1 TeV, the Higgs sector becomes non-perturbative and we must be concerned about violations of unitarity[39].

8 Kaluza-Klein Excitations Associated with Higher Dimensions

In our D-dimensional gauge theory, we must concern ourselves with the restrictions imposed on the scale size, Λ , and the compactification size, R , imposed by Kaluza-Klein excitations associated with the extra dimensions. Recently, Nath and Yamaguchi[40], and others[41], have studied the constraints on the compactification scale imposed by the Fermi constant, the fine structure constant, and the W and Z masses. Because of the importance of this issue for our theory, we shall discuss these results here in some detail. Dienes et al. have calculated the effects of infinite Kaluza-Klein towers on the "running" of the gauge coupling constants[7]. Let us now adapt their results to our FQFT formalism.

As before, we shall treat our four-dimensional space as an approximately flat Minkowski spacetime, and evaluate the vacuum polarization diagram, including the effects of the Kaluza-Klein excitations on the loop graphs. Consider first the case of a single Dirac fermion. We can generalize this result to the case of a realistic chiral fermion model as a final step. In FQFT, the vacuum polarization tensor is the sum of three parts, $\Pi_1^{\mu\nu}(p)$, $\Pi_2^{\mu\nu}(p)$, and

$\Pi_3^{\mu\nu}(p)$ corresponding to the standard loop graph, a tadpole graph and a contribution from the invariant measure factor. We have

$$\Pi_1^{\mu\nu} = \Pi_1^T(p^2) \left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) + \Pi_1^L(p^2) \frac{p^\mu p^\nu}{p^2}, \quad (77)$$

where Π^T and Π^L denote the transverse and longitudinal parts, respectively. The total transverse part is given by[11]:

$$\frac{\Pi^T(p^2)}{p^2} = -\frac{e^{(4)2}}{\pi^2} \sum_{n_i=-\infty}^{\infty} \exp\left(-\frac{p^2}{\Lambda^2}\right) \int_0^{1/2} dx x(1-x) E_i \left[x(1-x) \frac{p^2}{\Lambda^2} + \frac{m_n^2}{\Lambda^2} \right], \quad (78)$$

where $e^{(4)}$ is the four-dimensional coupling constant and

$$\sum_{n_i=-\infty}^{\infty} \equiv \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \dots \sum_{n_d=-\infty}^{\infty},$$

describes a summation over all Kaluza-Klein excitations with masses m_n , and m_0 is the energy of the ground state. Moreover, we see that $\Pi(p^2)$ vanishes exponentially fast in the Euclidean momentum region as $p^2 \rightarrow \infty$.

The sum over the Kaluza-Klein states can be performed by using the Jacobi θ_3 function[7]:

$$\theta_3(\tau) \equiv \sum_{n_i=-\infty}^{\infty} \exp(\pi i \tau n^2),$$

where τ is a complex number. This function obeys the property

$$\theta_3(-1/\tau) = \sqrt{-i\tau} \theta_3(\tau),$$

where the branch of the square root with non-negative real part is chosen. We obtain

$$\frac{\Pi(p^2)}{p^2} = -\frac{e^{(4)2}}{\pi^2} \exp\left(-\frac{p^2}{\Lambda^2}\right) \int_0^{1/2} dx x(1-x) E_i \left[x(1-x) \frac{p^2}{\Lambda^2} \right] \left[\theta_3\left(\frac{it}{\pi R^2}\right) \right]^d.$$

We then get

$$\left[\frac{\Pi(p^2)}{p^2} \right]_{p^2=0} = -\frac{e^{(4)2}}{12\pi^2} \int_{r^2/\Lambda^2}^{r^2/\mu_0^2} \frac{dt}{t} \left[\theta_3\left(\frac{it}{\pi R^2}\right) \right]^d, \quad (79)$$

where

$$r^2 = \frac{\pi}{(X_d)^{2/d}}$$

and

$$X_d = \frac{\pi^{d/2}}{\Gamma(1 + d/2)}$$

relates our FQFT scale Λ to the underlying physical mass scales. It is to be noted that in contrast to the standard cut-off technique, the FQFT calculation of the vacuum polarization is fully gauge invariant and unitary and Λ represents a *physical* scale in the theory.

For our full chiral gauge theory assuming that the Kaluza-Klein excitations arise from the gauge bosons and Higgs fields only, we obtain

$$\begin{aligned} \left[\frac{\Pi(p^2)}{p^2} \right]_{p^2=0} &= \frac{e_i^{(4)2}(b_i - \tilde{b}_i)}{8\pi^2} \ln\left(\frac{\Lambda}{\mu_0}\right) \\ &+ \frac{e_i^{(4)2}\tilde{b}_i}{16\pi^2} \int_{r^2/\Lambda^2}^{r^2/\mu_0^2} \frac{dt}{t} \left[\theta_3\left(\frac{it}{\pi R^2}\right) \right]^d, \end{aligned} \quad (80)$$

where the b_i are the one-loop beta-functions for the zero modes, while the \tilde{b}_i denote the beta-functions associated with the Kaluza-Klein excitations.

We can now obtain the scaling behaviour of our gauge coupling constants

$$\begin{aligned} \alpha_i^{-1}(\Lambda) &= \alpha_i^{-1}(\mu_0) - \left(\frac{b_i - \tilde{b}_i}{2\pi} \right) \ln\left(\frac{\Lambda}{\mu_0}\right) \\ &- \frac{\tilde{b}_i}{4\pi} \int_{r^2/\Lambda^2}^{r^2/\mu_0^2} \frac{dt}{t} \left[\theta_3\left(\frac{it}{\pi R^2}\right) \right]^d, \end{aligned} \quad (81)$$

where $\alpha_i \equiv e_i^{(4)2}/4\pi$. This gives the running of the gauge coupling constants as obtained by Dienes et al.[7]. The important result emerges that the Kaluza-Klein excitations convert the standard logarithmic scaling of the gauge coupling constants to a power law running behaviour.

By imposing matching conditions that the uncorrected value of the effective four-dimensional coupling constant α_i must agree with the value of the four-dimensional coupling $\alpha_i(\mu_0)$ at the scale μ_0 , we get

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln\left(\frac{\Lambda}{M_Z}\right) + \frac{\tilde{b}_i}{2\pi} \ln\left(\frac{\Lambda}{\mu_0}\right) - \frac{\tilde{b}_i X_d}{2\pi d} \left[\left(\frac{\Lambda}{\mu_0}\right)^d - 1 \right], \quad (82)$$

where M_Z is the mass of the Z-boson. This result is valid for all $\Lambda \geq \mu_0$. The Kaluza-Klein excitations cause *an acceleration of the unification of the gauge coupling constants* α_i ; this leads to the possibility of having gauge coupling unification at energies well below the usual GUT scale of 10^{16-19} GeV.

Let us consider the unregulated interaction Lagrangian, describing the coupling of fermions to zero modes and to the Kaluza-Klein modes:

$$\mathcal{L}_{\text{int}} = e_i^{(4)} J^\mu (A_{\mu i} + \sqrt{2} \sum_{n=1}^{\infty} A_{\mu i}^{(n)}), \quad (83)$$

where $A_{\mu i}$ are the zero modes and $A_{\mu i}^{(n)}$ are the Kaluza-Klein modes, respectively. Integrating out the W boson and its Kaluza-Klein excitations gives for the effective standard model Fermi constant[40, 41]

$$G_F^{\text{eff}} = G_F^{\text{SM}} K_d \left(\frac{M_W^2}{M_R^2} \right), \quad (84)$$

where $d = D - 4$ and K_d is

$$K_d(s) = \int_0^\infty dt \exp(-t) \left[\theta_3 \left(\frac{it}{s\pi} \right) \right]^d.$$

This integral diverges for $d > 1$, but a convergent result is obtained in regularized FQFT, corresponding to a truncation of the Kaluza-Klein states when the masses exceed the FQFT scale Λ , associated with the regulated Lagrangian.

Nath and Yamaguchi obtain the ratio of the Kaluza-Klein contribution to the Fermi constant to the standard model value of the Fermi constant for $d \geq 3$:

$$\frac{\Delta G_F^{KK}}{G_F^{\text{SM}}} \simeq \left(\frac{d}{d-2} \right) \frac{\pi^{d/2}}{\Gamma(1+d/2)} \left(\frac{\Lambda}{M_R} \right)^{d-2} \left(\frac{M_W}{M_R} \right)^2. \quad (85)$$

The gauge coupling evolution constrains M_R and Λ for TeV scale unification. Let us adopt the evolution equation[7] used by Nath and Yamaguchi:

$$\alpha_i(M_Z) = \frac{1}{\alpha_U} + \frac{b_i}{2\pi} \ln \left(\frac{M_R}{M_Z} \right) - \frac{b_i^{KK}}{2\pi} \ln \left(\frac{\Lambda}{M_R} \right) + \Delta_i, \quad (86)$$

where α_U is the effective GUT coupling constant, $b_i = (-3, 1, 33/5)$ for $SU(3)_c \times SU(2) \times U(1)$, $b_i^{KK} = (-6, -3, 3/5)$ are the b_i minus the fermion sector contribution which has no Kaluza-Klein excitations, and Δ_i are the

Kaluza-Klein corrections. Given d and M_R the unification of α_1 and α_2 fixes Λ/M_R . For the cases of d equal to 2, 3 and 4 extra dimensions, the Nath-Yamaguchi analysis produces the lower limits on M_R of 3.5 TeV, 5.7 TeV and 7.8 TeV. Thus, the observation of Kaluza-Klein excitations may be possible at the Large Hadron Collider (LHC).

Nath and Yamaguchi[40] have also considered the constraints arising from an analysis of $g_\mu - 2$ for extra d dimensions. These constraints on M_R at the 2σ level are $M_R > 1.6$ TeV for $d = 1$, $M_R > 3.5$ TeV for $d = 2$, $M_R > 5.7$ TeV for $d = 3$ and $M_R > 7.8$ TeV for $d = 4$.

9 Proton Decay Lifetime and Unified Theory Phenomenology

The problem of proton decay must be considered in the context of the unified models. When both colour triplet quarks and colour singlet leptons are assigned to the same irreducible representation of a symmetry group, then there exist vector bosons (leptoquarks) that transform leptons into quarks. Unless there exists a conserved quantum number A for which the proton is the lowest mass state with $A = 1$, then the proton can decay. We must guarantee within our unified field theory that the proton is stabilized sufficiently to not disagree with the experimental bounds on its lifetime, $\tau_p \geq 10^{32}$ yrs. For example, as is well-known, the conventional $SU(5)$ model of Georgi and Glashow[24] has been eliminated by the experimental bound on the decay rate. The problem becomes much more severe when we contemplate a compactification scale M_c of order 1-10 TeV.

In our theory, there exist several energy scales to consider. There is the Yang-Mills scale Λ_{YM} , associated with the Yang-Mills Lagrangian, \mathcal{L}_{YM} , the Higgs scalar field scale Λ_H , and the gravitational scale Λ_G . In addition, there is the aforementioned compactification scale M_c . Let us first consider the possibility that the Yang-Mills and the compactification scales are large, $\Lambda_{YM} \sim M_c \geq 10^{16}$ GeV.

For the $SO(10)$ model, we choose the left-right symmetric $SU(2)$ Pati-Salam breaking pattern [42]:

$$\begin{aligned} SO(10) &\rightarrow SU(4) \times SU(2)_L \times SU(2)_R \rightarrow SU(3) \times SU(2)_L \times SU(2)_R \times U'(1)_{B-L} \\ &\rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)_{em}. \end{aligned}$$

Here, the sequence of characteristic Higgs potentials are described by the representations [54](#), [45](#), [16](#) and [10](#). At one loop level, the renormalization group equations are for $\mu < M_c \sim \Lambda_{\text{YM}}$ [7, 43]

$$\sin^2 \theta_W(M_W) = \frac{3}{8} - \frac{11}{3} \frac{\alpha(M_W)}{\pi} \left[\frac{5}{8} \ln \left(\frac{M_U}{M_W} \right) - \frac{3}{8} \ln \left(\frac{M_U}{M_S} \right) \right], \quad (87)$$

and

$$\frac{\alpha_{\text{em}}(M_W)}{\alpha_3(M_W)} = \frac{3}{8} - \frac{11}{8} \frac{\alpha(M_W)}{\pi} \left[3 \ln \left(\frac{M_U}{M_W} \right) - \ln \left(\frac{M_U}{M_S} \right) \right], \quad (88)$$

where M_U denotes the mass of the exchanged boson that breaks $SO(10)$, while M_W is the weak scale mass. We have assumed that $M_C \sim M_U$, where M_C is the mass of the exchanged boson that breaks $SU(4) \times SU(2)_L \times SU(2)_R$ to $SU(3) \times SU(2)_L \times SU(2)_R \times U'(1)_{B-L}$. Moreover, M_S denotes the mass associated with the breaking of $SU(3) \times SU(2)_L \times SU(2)_R \times U'(1)_{B-L}$ to $SU(3) \times SU(2) \times U(1)$. The term proportional to $\ln(M_U/M_W)$ corresponds to the low energy group $SU(3) \times SU(2) \times U(1)$. We can choose M_S to keep the prediction for $\alpha(M_W)/\alpha_3(M_W)$ fixed at the $SU(5)$ value, while varying M_U from the usual $SU(5)$ value for M_X . This increases $\sin^2 \theta_W(M_W)$ by approximately $0.005 \ln(M_U/M_X)$. We can still obtain reasonable values for $\sin^2 \theta_W$ for a large variation of M_U . Consequently, the proton decay lifetime, which satisfies

$$\tau_p \propto 10^{30} \text{ yrs} \left(\frac{M_U}{5 \times 10^{14} \text{ GeV}} \right)^4 \quad (89)$$

can be made to satisfy the experimental bounds.

This illustrates the freedom in building an $SO(10)$ model, because of the uncertainties in the values of M_U and τ_p , once we go beyond the simplest, minimal symmetry breaking scheme. However, these models do possess the possibility of having very light scales of intermediate unification, which may be associated with baryon and lepton number violating processes in other than proton channel decays.

We must now turn our attention to the Higgs hierarchy problem. With the foregoing scenario, we can choose for the Higgs scale, $\Lambda_H \sim 1 \text{ TeV}$, which guarantees in FQFT (see Sect.7) that the *radiative loop contributions to the scalar Higgs self-energy are controlled*, leaving the tree graphs untouched at higher energies, which can mediate Higgs spontaneous symmetry breaking. We have the possibility of reducing the graviton quantum loop contributions

to the TeV scale by choosing $\Lambda_G \sim 1$ TeV, or choosing a much higher value for Λ_G , e.g. the Planck scale $\sim 10^{19}$ GeV.

This scenario does solve the Higgs hierarchy problem, while simultaneously ensuring a sufficiently stable proton, but the extreme difference between the Yang-Mills scale Λ_{YM} and the Higgs scalar field scale Λ_H does raise questions about "naturalness". However, we stress that these energy scales are associated with *physical* scales in FQFT, and not with arbitrary cut-offs.

Let us consider next a scenario in which $\Lambda_{YM} \sim M_c \sim \Lambda_H \sim \Lambda_G \sim 1$ -10 TeV. This produces, at first sight, an attractive physical picture in which we can contemplate observing new physics with the next generation of accelerators. The classical GR theory is left unchanged, because only the graviton quantum loop graphs are reduced to a scale of 1-10 TeV, leaving classical GR intact to all energies. The gauge coupling constant will satisfy a power law behaviour when $\Lambda_{YM} \sim M_c > \mu$, so gauge coupling unification is accelerated, as explained in Sect 8. There is no Higgs hierarchy problem in four dimensions, and for our $SO(18)$ unification scheme *we obtain the correct prediction of three chiral families and the standard model in four dimensions*. However, we are faced with the *bête noire* of potentially fast proton decay.

There are two possible scenarios that we can adopt to circumvent the problem of baryon and lepton number violation. The first is to introduce a global $U(1)$ symmetry. The conservation of A does not carry with it a long-range force, so even though A is an additive quantum number like charge Q , it cannot be a generator of a local $U(1)$. However, if a global $U(1)$ and a local $U(1)$ are both broken in such a way that a linear combination of the generators is conserved, then the vector boson acquires a mass and the unbroken linear combination yields an exact conservation law[44]. Let X be the generator of the local $U(1)$ and Z the generator of the global $U(1)$. We choose $Z = 0$ for the fermions and assign all fermions to a single irreducible representation (as is the case for our $SO(10)$ in four dimensions). The Higgs fields have non-zero values of Z , and if Z and X are broken and their sum is conserved, then some of the scalar fields will acquire non-zero A , resulting in a heavy boson and $A = X$ in the fermion sector. Thus, our $SO(10)$ theory has a stable proton. It is possible to break the Z and X so that no "weird" fermions exist, but in general there will exist "weird" fermions at higher energies.

Of course, the introduction of a global $U(1)$ symmetry into the theory is considered by some to be an unpalatable way of solving the proton decay problem. It goes against the spirit of local gauge symmetries. It means that

if we rotate a proton in our living room, then exactly the same rotation is required to be performed in the Andromeda galaxy. There is also the danger that gravitational interactions will induce fast proton decay in higher-dimensional operators. Moreover, there is the more abstract issue that black holes violate all non-gauged symmetries.

Another possible scenario to evade the proton decay issue has recently been proposed by Arkani-Hamed and Schmaltz[45]. They “stick” the standard model fermions at different points on domain walls in the extra d dimensions. The couplings between them are suppressed due to the exponentially small overlaps of their wave functions. We can adopt this mechanism in our higher-dimensional field theory. The model can be simply visualized in one extra dimension, in which the gauge fields and the Higgs fields are allowed to propagate inside the wall, while the fermions are constrained to different points in the wall. The fermion wave functions are described by narrow Gaussian functions. The long-distance four-dimensional theory can have exponentially small Yukawa couplings, generated by the small overlap between left- and right-handed fermion wave functions. This can lead to an exponentially suppressed proton decay rate, if the quarks and leptons are localized to separate ends of the wall. This avoids the issue of inventing symmetries in the theory which protect the proton from a fast decay rate. Since this mechanism relies only on the dynamics of the wall geometry and the placements of the fermions on the walls in the extra dimensions, one may develop an uneasy feeling that the mechanism is somewhat contrived, but at present there appears to be no obvious critical reason why this could not be an acceptable way to resolve the proton decay problem.

10 Conclusions

A higher-dimensional unified field theory based on a Kaluza-Klein-Yang-Mills-Higgs action and a ground state $M^4 \times B$ is developed. The gauge group has a topologically non-trivial sub-group associated with the compact internal space, i.e. a Dirac monopole is inserted into the latter group to guarantee that the compactification to four dimensions retains the chiral non-vector-like property of the fermions. We choose the minimal, anomaly free model $SO(18)$ in twelve dimensions with the breaking to $SO(8) \times SO(10)$, leading to a four-dimensional group $SO(10)$ GUT model, which contains the standard model $SU(3)_c \times SU(2) \times U(1)$ and predicts *three families of quarks*

and leptons. The chiral nature of the fermion representations is guaranteed in the dimensional reduction process by making $SO(8)$ topologically non-trivial with the reduction $SO(8) \rightarrow Sp(4) \times SU(2)$. The fermions have a non-vanishing chirality number, whereby the left-handed quarks and leptons have the correct physical interpretation.

The problem of the stability of Kaluza-Klein theories can be solved in our model by means of the supplementary Yang-Mills fields or the magnetic monopole that exists in the internal space, which can generate the repulsive forces necessary to balance the gravitational forces.

The important gauge hierarchy problem, associated with the Higgs sector, is solved by the exponential damping of the Higgs self-energy in the Euclidean p^2 domain for $p^2 > \Lambda_H^2$, and for a Λ_H scale in the TeV range.

The unified field theory will have three coupling constants, namely, the gravitational constant \bar{G} , the gauge coupling constant e , and the scalar Higgs coupling constant g , which have to be rescaled in four dimensions. Dimensional reduction to four dimensions can explain charge quantization in terms of the compactification scale R . In string theory, the coupling constants are determined by the string dilaton scalar field, so in principle there are no arbitrary coupling constants. However, this presupposes that one knows the *complete* solution to the dynamics of the string equations. It is difficult to see how the determination of the coupling constants in string theory can be implemented in practice, particularly, since it would appear that only non-perturbative solutions can be obtained in M-theory.

The critical issue of the finiteness of quantum gravity perturbation theory in D dimensions is solved by applying the FQFT formalism. The nonlocal quantum loop interactions reflect the non-point-like nature of the field theory, although we do not specify the nature of the extended object that describes a particle. Thus, as with string theories, the point-like nature of particles is “fuzzy” in FQFT for energies greater than the scale Λ . One of the features of superstrings is that they provide a mathematically consistent theory of quantum gravity, which is ultraviolet finite and unitary. FQFT focuses on the basic mechanism behind string theory’s finite ultraviolet behavior by invoking a suppression of bad vertex behavior at high energies, without compromising perturbative unitarity and gauge invariance. FQFT provides a mathematically consistent theory of quantum gravity at the perturbative level. If we choose $\Lambda_G \sim 1\text{-}10$ TeV, then quantum radiative corrections to the classical tree graph gravity theory are perturbatively negligible to all energies greater than Λ_G including the Planck energy. If, on the other hand, we

choose $\Lambda_G \sim M_{\text{Planck}}$, then we are forced to seek a non-perturbative FQFT quantum gravity formalism at the Planck scale.

Our solution of the finiteness of quantum gravity is based on a *gauge field theory*, which allows us to go off the mass shell when calculating vertex operators. It is generally accepted that there is no self-consistent string field theory, because such a theory would correspond to a ϕ^3 theory with a Hamiltonian unbounded from below, causing the field theory to be unstable. Our FQFT can be a stable theory as was shown by Kleppe and Woodard[14]. However, our higher-dimensional field theory may be linked to a final, realized stable version of M-theory, for the finiteness of FQFT owes its existence to an ultraviolet suppression mechanism akin to that of string theory, and a field theory version of M-theory could possess a structure similar to our FQFT. Superstring theory and its offspring appear to lead to fundamental changes in our understanding of space and time[46]. In particular, our everyday notions of causality may be altered at high energies and small distances. Our introduction of nonlocal interaction Lagrangians in FQFT may be the modification of local, point particle field theory at the quantum level that is needed to achieve a mathematically consistent theory of quantum gravity.

Supersymmetry is required if we wish to unify the particle spins of bosons and fermions. This would be a mathematically beautiful achievement. We could incorporate supersymmetry in our higher-dimensional FQFT, and indeed in contrast to, for example, dimensional regularization, FQFT respects continuous supersymmetry gauge transformations to all orders of perturbation theory. However, supersymmetry partners have not, as yet, been observed and as we have demonstrated, FQFT can resolve the Higgs hierarchy problem without supersymmetry, thereby removing the primary reason for promoting supersymmetry at the phenomenological level.

Because we are able to lower the compactification scale M_c to the TeV energy range, we anticipate that Kaluza-Klein excitations will be observable at these energies by the LHC. In order to distinguish these Kaluza-Klein signatures from the signatures of supersymmetry partners or other exotic physics, we must analyze further the decay properties of the Kaluza-Klein modes, so as to select possible unique features associated with the excitations.

We have adopted the idea of reducing quantum gravity to lower energies by choosing the quantum gravity scale Λ_G in the TeV range, or even at much lower energies. This is in accord with the recent interesting idea that the gravitational scale could be in the TeV range[8]. However, in contrast to the work in ref. (8), we only lower the energy scale of the quantum gravity loops

through the choice of the scale Λ_G , without affecting the classical tree graphs which sum to give local and causal, classical Newtonian and GR theories.

We have not considered the implications of our theory for cosmology and black holes, nor have we concerned ourselves with the important problem of the cosmological constant. These issues will be addressed in future work.

Acknowledgments

This work was supported by the Natural Sciences and Engineering Research Council of Canada.

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